Section 5.1 – Sets and Set Operations

A collection of objects is called a **set**.

An object of a set is called an **element**.

Notation:
$\in$ = "element of"
$\notin$= "not an element of"

Example 1: Let B = {a, b, c, …, y, z}. In **set-builder notation,** the set B can be written as follows:

Equality of Sets
Let A and B be two sets. We say that A is equal to B, written as A = B. This is true if and only if A and B have exactly the same elements. If two sets are not equal we write A $\neq$B.

Subsets
Let A and B be two sets. We say that A is a subset of B or that is contained in B and written A $\subseteq$B. From the definition it follows that for any set A, A $\subseteq$A; that is, every set is a subset of itself.

Proper Subsets
If $A \subseteq B$, but $A \neq$B then A is a **proper subset** of B. If A is a proper subset of B then we write $A \subset B$
In other words: A is a proper subset of B if the following two conditions hold.

1. A $\subseteq$B
2. There exist at least one element in B that is not in A.

Example 2: Let A ={1,2,3}, B = {1,2,3,4,5}and C = {3,2,1} . In the following, answer true or false in the following:

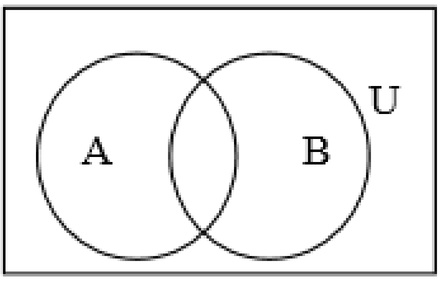
A = C T or F
A $\subseteq$ C T or F
A $\subset$B T or F
C $\subset$A T or F
5 $\notin$ C T or F

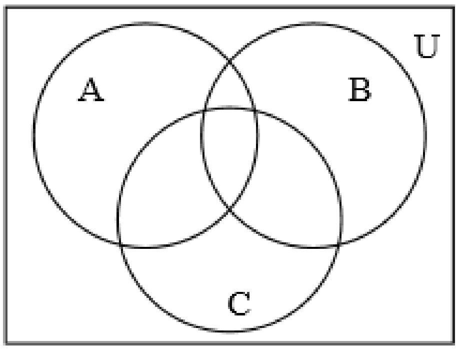
A set that contains no elements is called the **Empty Set**
Note: We write Ø to denote the empty set. The symbol Ø is a subset of every set.

Example 3: Let A = {a, b, c}. List all subsets and proper subsets of the set A.

The **Universal set** is the set of interest in a particular discussion.

A **Venn diagram** is a visual representation of sets.
They look like:



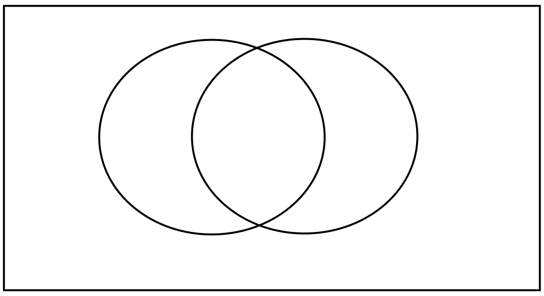


Set Operations

Let A and B be two sets. The set of all elements that that belong to either A *or* B or both is called the **Union** of A and B (denoted A ∪ B).

In set builder notation $A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or both}\}$

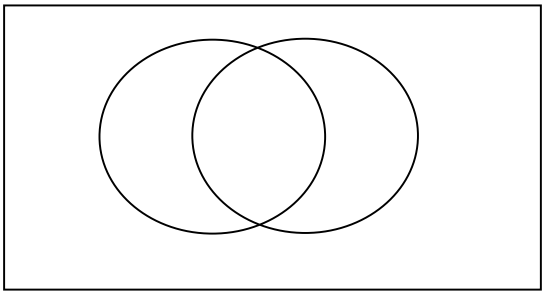
Set Union in a Venn diagram looks like:



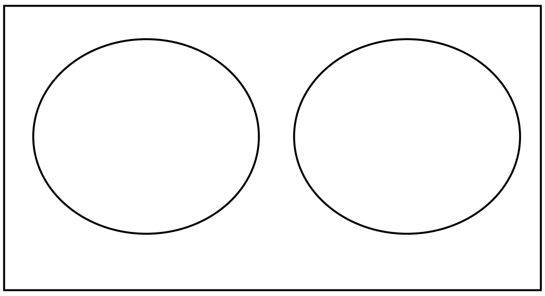
Let A and B be two sets. The set of all elements in common with both sets A *and* B is called the **Intersection** of A and B (denoted $A \cap B$).

In **set-builder notation** $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Set Intersection in a Venn diagram looks like:



If $A \cap B = \emptyset$, then we say the intersection is the **null intersection** and that A and B are **disjoint**.

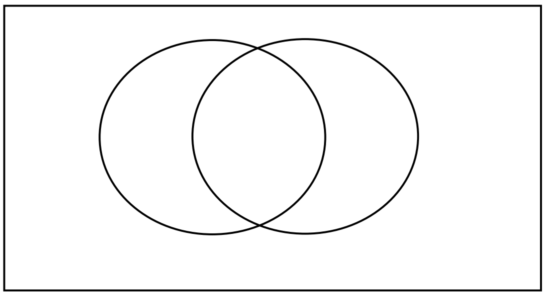


Let U be a universal set and $A \subseteq U$. The set of all elements in U that are not in A is called the

Complement of A. (denoted A^C)

In **set-builder notation** $A^c = \{x \mid x \in U, x \notin A\}$

Set Complementation in a Venn diagram looks like:



Set Operations
Let U be a universal set and A and B be subset of U

$U^c = \emptyset$ $\emptyset^c = U$ $(A^c)^c = A$

$A \cup A^c = U$ $A \cap A^c = \emptyset$

$A \cup B = B \cup A$ $A \cap B = B \cap A$

DeMorgan's Laws

$A^C \cap B^C = (A \cup B)^C$

$A^C \cup B^C = (A \cap B)^C$

Example 3: Let U={1,2,3,4,5,6,7,8,9,10}

A={1,3,5,7,9}

B={2,4,6,8,10}

C={1,2,4,5,8}

Find the given sets.
a. $(A \cup B)$

b. $(B \cap C)$

c. $(B \cap C^c)$

d. $(A \cup B \cup C)^c$

e. $A \cup (B^c \cap C)$

f. $(A^c \cap B^c) \cup C$

Example 4: Let U denote the set of all employees at a certain Company.
Let T={$x \in$ U| x likes to read Time magazine}, E={$x \in$ U| x likes to read ESPN magazine} and C={$x \in$ U| x likes to read Car and Driver}.

Part A. Describe the given set in words given statement in set notation.

i. T ∪ C = the set of all employees at this company that

ii. $(T^c \cap C) \cup E$ = the set of all employees at this company that

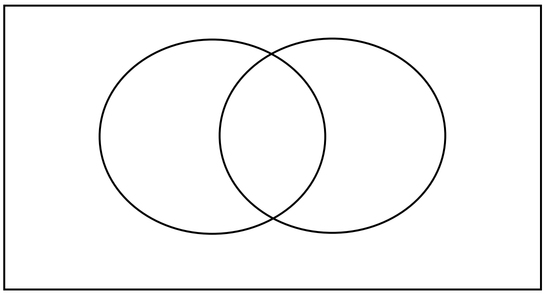
Part B. Describe the given statement in set notation.

i. The set of all employees at this company that like ESPN and do not like Car and Driver.

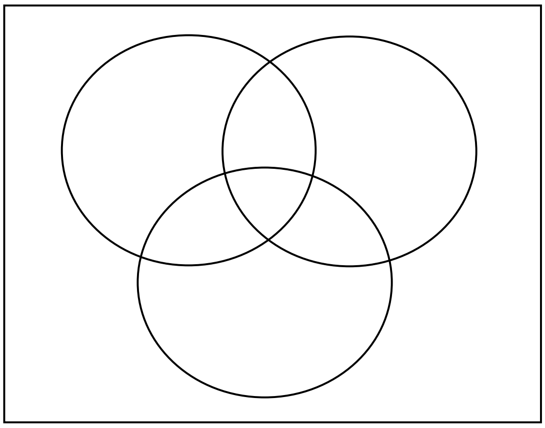
ii. The set of all employees at this company that do not like Time, ESPN or Car Driver.

Another good example is example 8 and 9 in your book. Read through that example.

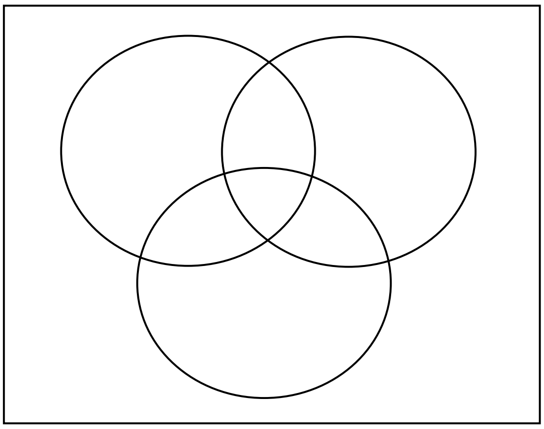
Example 5: Shade the portion of the Venn diagram that represents the given set.
(Assume the given sets are not disjoint.)
a. $(A \cap B^c)$



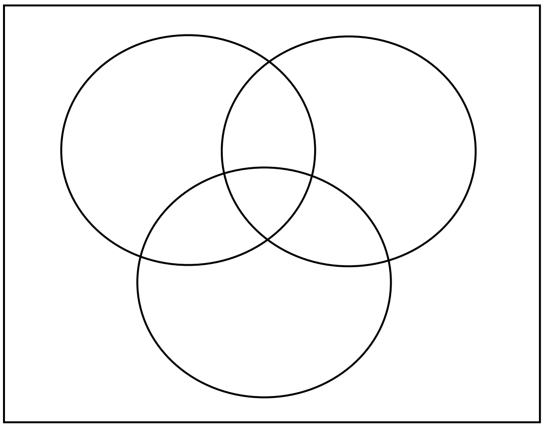
b. $A \cup (B \cap C)$



c. $(B \cap C)^{c} \cap A^{c}$



d. $A^{c} \cap (B\ U\ C\ )$



e. $(C^{c} \cup\ B^{c} \cup A^{c}\ )^{c}$